Minicourse

Long Time Dynamics on the Incompressible Euler Equations

Speaker: In-Jee Jeong (Seoul National University)

Location: Ding Shisun Lecture Hall, Zhihua Building（智华楼一层 111 丁石孙教室）

Time: June 19 (Thursday), 9:30am -- 11:30am

June 20 (Friday), 9:30am -- 11:30am

June 21 (Saturday), 2:00pm -- 4:00pm

Lecture 1 (June 19, 9:30am -- 11:30am)

Title: *Vortex stability for 2D incompressible Euler equations*

Abstract: Large-scale coherent vortex structures are frequently observed in nature, and their existence and stability can be attributed to the variational principle involving the kinetic energy of the fluid. Starting with the simplest example of the Rankine vortex patch, we review several classical settings where vortex stability is obtained by formulating a kinetic energy variational principle with appropriate constraints. The examples include Kirchhoff ellipses and the Lamb dipole.

Lecture 2 (June 20, 9:30am -- 11:30am)

Title: *Stability and small scale creation for 2D incompressible Euler equations*

Abstract: Numerical and experimental studies of the 2D Euler equations show formation of complex vortex structures and in particular a lot of small scale features. The small scale creation can be quantified for instance using the growth of the vorticity gradient in the maximum norm. We shall review various benchmark works obtaining lower bounds on the vorticity gradient, which diverges as time goes to infinity. Somewhat counterintuitively, proving stronger gradient growth requires having a stronger sense of vortex stability. We also discuss various open problems in this direction.

Lecture 3 (June 21, 2:00pm -- 4:00pm)

Title: *Vortex confinement problem for axisymmetric incompressible Euler equations*

Abstract: The three-dimensional incompressible Euler equations under axisymmetry have been widely studied. While the “no-swirl” assumption makes the system very similar to the two-dimensional vorticity equations, it is still possible for solutions to have unbounded vortex stretching. After reviewing classical confinement results in 2D and 3D, we report some progress on the issue of vortex stretching for Euler equations under rotational symmetries in three dimensions.