Mathematical Control Day

In this workshop, we share some recent developments of mathematical control theory for PDEs and SDEs. The main goal of this event is to bring together experts in control theory from different perspectives.

Time and Venue:

April 10, 2025, Siyuan Hall, Zhihua Building (智华楼四元厅), Peking University

Institution：

Peking University & Shandong University

Invited speakers:

Jean-Michel Coron (Sorbonne Université)
Bao-Zhu Guo (Chinese Academy of Sciences)
Shige Peng (Shandong University)
Gengsheng Wang (Tianjin University)
Xu Zhang (Sichuan University)

Time	Titles	Speakers
8:45-9:00	Opening Session	
9:00-10:00	G-expectation weighted Sobolev spaces, backward SDE and path dependent PDE	Shige Peng
10:00-10:30	Tea break	
10:30-11:30	Controlled flows of maps from the circle to compact Riemannian manifolds	Jean-Michel Coron
11:30-13:30	Lunch	
13:30-14:30	The H-infinity control of parabolic PDEs	Bao-Zhu Guo
14:30-14:45	Tea break	
14:45-15:45	Observability inequality, log-type Hausdorff content and heat equations	Gengsheng Wang
15:45-16:00	Tea break	
16:00-17:00	Can nonlinear PDEs and their control be replaced by some linear versions?	Xu Zhang
17:00	Summary and Conclusions	

Titles and Abstracts

Controlled flows of maps from the circle to compact Riemannian manifolds

Jean-Michel Coron (Sorbonne Université)

Abstract: In this presentation, we explore recent developments concerning the evolution of maps from the circle into compact Riemannian manifolds, with a particular focus on their dynamics in relation to the Dirichlet energy. Specifically, we examine two fundamental flow equations: the wave maps equation and the heat maps equation. By introducing a localized control force into these flows, we investigate key questions of controllability and stabilization, shedding light on the extent to which these systems can be influenced or guided towards desired states. We shall emphasize the interplay between geometric analysis, partial differential equations, and control theory.

This talk is based on the following articles:

1. Joachim Krieger and Shengquan Xiang, Semi-global controllability of a semilinear wave equation, 2022.
2. JMC, Joachim Krieger, and Shengquan Xiang, Global controllability and stabilization of the wave maps equation from a circle to a sphere, 2023.
3. JMC and Shengquan Xiang, Global controllability to harmonic maps of the heat flow from a circle to a sphere, 2024.
4. JMC, Joachim Krieger, and Shengquan Xiang, Global controllability of the wave maps equation from a circle to a Riemannian manifold, 2025

Biography: Jean-Michel Coron is an Emeritus Professor at Sorbonne University and an Honorary Professor at Sichuan University. His research includes geometric analysis, partial differential equations, and nonlinear control theory. He has created a school in control theory by the supervision of an important number of first class students who are now at the top of the field worldwide.

Jean-Michel Coron was awarded numerous prizes, such as Peccot lecture (1987), Fermat prize (1993), ICIAM Maxwell prize (2015), Reid prize (2017). He was a plenary speaker at the ICM 2010 and ICIAM 2015, and an invited speaker at ICM 1990. He is a member of French Academy of Sciences (2014) and a member of Academia Europaea (2021).

The H-infinity control of parabolic PDEs

Bao-Zhu Guo (Chinese Academy of Sciences)

Abstract: H-infinity control represents a significant paradigm shift in robust control research in the 1980s, providing a systematic approach to studying disturbances both within and outside the system. In particular, model approximation and output tracking can be formulated as standard H-infinity control problems for solution. For linear, finite-dimensional, time-invariant systems, H-infinity control can be derived using several methods that yield the same conclusion and can be solved analytically: either by solving the matrix Riccati equation to obtain the optimal control and worst-case disturbance, both in the form of linear feedback, or by solving matrix inequalities.

However, the Riccati equation corresponding to partial differential equation systems (PDEs) is an infinite-dimensional operator equation, making analytical solutions almost impossible. Finding the optimal feedback control for PDE systems is a challenging problem. By combining methods for solving approximate solutions to infinite-dimensional LQ problems and finite-dimensional H-infinity control, we have obtained approximate solutions and convergence results for optimal state feedback in high-dimensional parabolic systems. This report presents some preliminary theories and results in this area.

Biography: Bao-Zhu Guo is a Research Professor at the Academy of Mathematics and Systems Science, Chinese Academy of Sciences. Formerly served as a Chair Professor of Computational and Applied Mathematics at the University of the Witwatersrand, South Africa.

Bao-Zhu Guo's main research area is control theory for distributed parameter systems. He has conducted systematic research on non-collocated design for PDE systems, Riesz basis theory, well-posedness and regularity of PDE systems, and numerical solutions for optimal control. He has also published five monographs in Springer, Wiley & Sons and Birkhäuser etc.

G-expectation weighted Sobolev spaces, backward SDE and path dependent PDE

Shige Peng (Shandong University)

Abstract: Beginning from a space of smooth, cylindrical and non-anticipative processes defined on a Wiener probability space, we introduce a P-weighted Sobolev space, or "P-Sobolev space"of non-anticipative path-dependent processes $V = V(t,\omega)$ such that the corresponding Sobolev derivatives $Dt + (1/2)\Delta$ and Dx are well defined on this space. We identify each element of this Sobolev space with the one in the space of classical Lp integrable Ito's process. Consequently, a new path-dependent Ito's formula is applied to all such Ito processes.

It follows that the path-dependent nonlinear Feynman-Kac formula is satisfied for most LP-solutions of backward SDEs: each solution of such BSDE is identified with the solution of the corresponding quasi-linear path-dependent PDE (PPDE). Rich and important results of existence, uniqueness, monotonicity and regularity of BSDEs, obtained in the past decades can be directly applied to obtain their corresponding properties in the new fields of Path-Dependent PDEs.

Moreover, we replace the linear Wiener expectation by a sublinear fully nonlinear G-expectation and then introduce the corresponding fully nonlinear G-expectation weighted Sobolev space, or "G-Sobolev space", in which the derivatives DtV, DV and D2V are all well defined separately. We then formulate the corresponding fully nonlinear Path-PDEs in the G-Sobolev space and then identify them to a type of backward SDEs driven by fully nonlinear G-Brownian motion.

Biography: Shige Peng is a chair professor in the School of Mathematics at Shandong University. His main researches are in the domains of stochastic optimal controls, backward stochastic differential equations and the corresponding partial differential equations, stochastic HJB equations. Recently he is interested in the theoretical foundation of nonlinear expectations.

Some awards he has received are the natural Science Prize of China (1995), Su Buqing Prize of Applied Mathematics (2006), Chinese Academy of Science TAN Kah Kee Science Award (2008), Chinese Society of Mathematics Hua Loo-Keng Award (2011), and the Qiu Shi Award for Outstanding Scientists (2016). He was a plenary speaker at the ICM 2010 and ICIAM 2015. He is a member of Chinese Academy of Sciences (2005) and a member of Academia Europaea (2023).

Observability inequality, log-type Hausdorff content and heat equations

Gengsheng Wang (Tianjin University)

Abstract: This paper studies the observability inequality for heat equations defined on a bounded domain of R^d and the whole space R^d respectively, where the observation sets are measured by a Hausdorff content, defined by a log-type gauge function, which is closely related to the heat kernel.

For the heat equation on a bounded domain, we obtain the observability inequality for observation sets of positive log-type Hausdorff content, which, in particular, implies the observability inequality for observation sets of positive s-dim Hausdorff measure, where s can be any number in (d-1,d]. For the heat equation over R^d, we build up the observability inequality for observation sets which are thick at scale of the log-type Hausdorff content. This is a recent work joint with H. Shanlin and M. Wang.

Biography: Gengsheng Wang is a Professor of Mathematics at Tianjin University. His main research interest lies in the field of the control theory of partial differential equations.

He has achieved important works concerning time optimal control, observability of the heat equations and Schrödinger equations. He has over one hundrad publications in prestigious mathematics journals including J. Eur. Math. Soc.、J. Math. Pures Appl.、SIAM J. Control Optim.、ESAIM Control Optim. Calc. Var. He serves as an editor of the journal SIAM J. Control Optim. and ESAIM Control Optim. Calc. Var.

Can nonlinear PDEs and their control be replaced by some linear versions?

Xu Zhang (Sichuan University)

Abstract: It is well-known that there exist huge challenges in nonlinear PDEs and their control. Naturally, people hope to replace them by some linear versions but most of these remain to be done, and therefore this direction is full of open problems. Particularly, a controlled semilinear parabolic equation can be formally replaced by its linear version, when their initial data and control actions are sufficiently small. In this work, we prove this fact and also establish a relationship on the null controllability of these equations. More precisely, we prove that both equations are not only null controllable, but the errors between their associated solutions are also a high-order infinitesimal with respect to their small initial values. This indicates that, in some sense, the study on null controllability of the semi-linear parabolic equation with small initial data can be indeed replaced by that of the linear problem. Nevertheless, so far we have not obtained the same result, either for quasilinear parabolic equation or for semi-linear hyperbolic equations.

Biography: Xu Zhang is a professor at Sichuan University. His research focuses on control theory and infinite dimensional analysis. His main works have been published in SIAM Rev., Comm. Pure App Math, Annu. Rev. Control, J Eur. Math Soc., Mem. Amer. Math. Soc., etc. He has also authored three monographs in Springer.

He has won the National Natural Science Award (2013), the SIGEST Paper Award from CSIAM in 2018. He has served as editorial board members for more than 10 international academic journals including SIAM J. Control Optim. He was an invited speaker at ICM 2010.